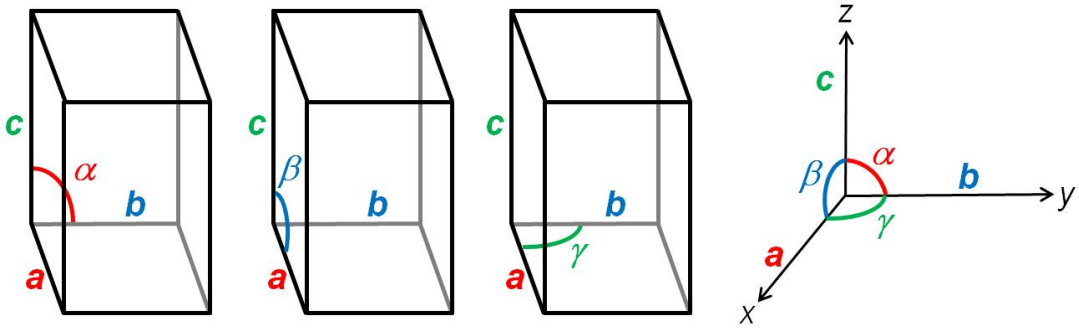
The seven crystal systems and the fourteen Bravais lattices[1]

In the crystalline solid state, periodicity is described by a three-dimensional repeat unit known as a *unit cell*. Although the unit cell for each crystalline material may be unique to that substance, unit cells can be classified by commonly observed features of structure and symmetry. The broadest classification of unit cells are the *seven crystal systems*. The *Bravais lattice* system considers additional structural details to divide these seven systems into 14 unique Bravais lattices. When the symmetry elements of the lattice structure are also considered, over 200 unique categories, called *space groups,* are possible.

The lattice parameters for a unit cell are referred to by a standard lettering system. The length, width, and height of the unit cell are called *a, b,* and *c* (the x, y, and z coordinate axes, respectively). The three angles that define the unit cell are called α (between *b* & *c*), β (between *a* & *c*), and γ (between *a* & *b*) (**Figure 1**).



In this exercise, you will use wooden sticks and candy to build examples of the seven crystal systems and the fourteen Bravais lattices. Use the pictures and descriptions below to build examples of each lattice. You will submit the models at the end of the exercise. You will also photograph or sketch your work.

Materials

7.5 oz box of DOTS gumdrops (©TRI, LCC)
wooden sticks, 3 different sizes: 12", 10", 6", or 4" bamboo skewers, and/or 2 5/8" toothpicks

The seven crystal systems

1, 2, 3. cubic, tetragonal, and orthorhombic. In these crystal systems, the angles α, β, and γ are all = 90°. What differentiates these three systems are the lattice parameters *a, b,* and *c.*

1. **Cubic.** Using gumdrops and wooden sticks, build <u>two</u> cubic crystals with ***a* = *b* = *c*, all angles = 90°**. Photograph or sketch the results.

2. **Tetragonal.** Using gumdrops and wooden sticks, build <u>two</u> tetragonal crystals with ***a* = *b* ≠ *c*, all angles = 90°.** Photograph or sketch the results.

Note that in the **tetragonal** and **cubic systems**, some of the cell lengths are equivalent (e.g, in the tetragonal system *a* = *b*). In this case, **a unique letter is not needed to describe the equivalent cell length** (since *a* = *b* in the tetragonal system, both the *a* and *b* sides of the unit cell are labelled "*a*").

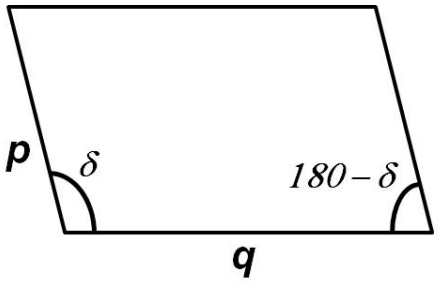
3. **Orthorhombic.** Using gumdrops and wooden sticks, build <u>three</u> orthorhombic crystals with ***a* ≠*b* ≠ *c*, all angles = 90°.** Photograph or sketch the results.

4, 5. monoclinic and triclinic. These two systems are similar to orthorhombic crystals in that the cell lengths *a, b,* and *c* are unique. However, one or more of the cell angles are distorted.

4. **Monoclinic.** Take one of the *orthorhombic* unit cells that you built and modify it so that ***a* ≠ *b* ≠ *c*, $\beta = \gamma = 90°$, $\alpha \neq 90°$.** Photograph or sketch the results.

5. **Triclinic.** Take another of the *orthorhombic* unit cells that you built and modify it so that ***a* ≠ *b* ≠ *c*, $\alpha \neq \beta \neq \gamma \neq 90°$.** Photograph or sketch the results.

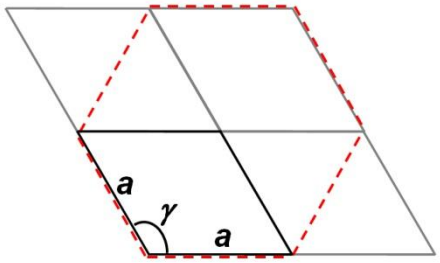
When representing a non-90° cell angle between two axes, there are two options. For example, examine the drawing of the parallelogram shown here (**Figure 2**). The angle δ describes one angle of a parallelogram formed by sides *p* and *q*. This parallelogram also contains a second angle with value 180° - δ. The convention in crystallography is usually to **choose the angle that is larger than 90°, while preserving a right handed coordinate system**.



6, 7. hexagonal and trigonal (rhombohedral subset). To fully appreciate the differences between these crystal systems, one must consider the symmetry elements of their respective lattices. Unfortunately, this is beyond the scope of this exercise.

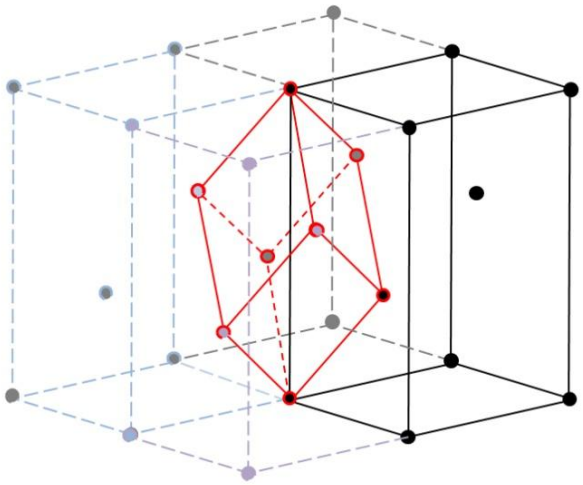
6. **Hexagonal.** Take one of the *tetragonal* unit cells that you built and modify it so that ***a* = *b* ≠ *c*, $\alpha = \beta = 90°$, $\gamma = 120°$.** Photograph or sketch the results.

The hexagonal unit cell resembles tetragonal crystals in that *a* = *b* (so like tetragonal systems, a separate *b* parameter is unnecessary), but angle γ = 120° instead of 90°. This picture on the left (**Figure 3**) shows several hexagonal cells viewed along the *c*-axis, illustrating the hexagonal nature of this system.



7. **Trigonal (rhombohedral subset).** Take one of the *cubic* unit cells that you built. Pull along the body diagonal to modify it so that ***a* = *b* = *c*, $\alpha = \beta = \gamma \neq 90°$.** Photograph or sketch the results.

This system is tricky. A **rhombohedral** cell resembles a cubic cell in that *a* = *b* = *c* and $\alpha = \beta = \gamma$, but the angle is NOT 90°. Rhombohedral crystals can actually be described two ways: with either a rhombohedral or a **trigonal** lattice (**Figure 4**), which resembles the hexagonal system (***a* = *b* ≠ *c***, $\alpha = \beta = 90°$, $\gamma = 120°$). The difference between trigonal and hexagonal systems is mainly the location of the lattice points (and therefore the positions of symmetry). For this reason, sometimes the "seven" crystal systems are simplified to six, and rhombohedral and trigonal lattices are simply called 'hexagonal'. However, when symmetry and lattice points are considered, hexagonal lattices really are different from trigonal lattices.



The fourteen Bravais lattices

The fourteen Bravais lattices begin to consider important locations within the crystal structure, called *lattice points*. An atom, collection of atoms, or symmetry operation(s) may lie on these lattice points within a crystal belonging to this class. The space group notation used in the *International Tables for Crystallography*[2] distinguishes between the different Bravais lattices.

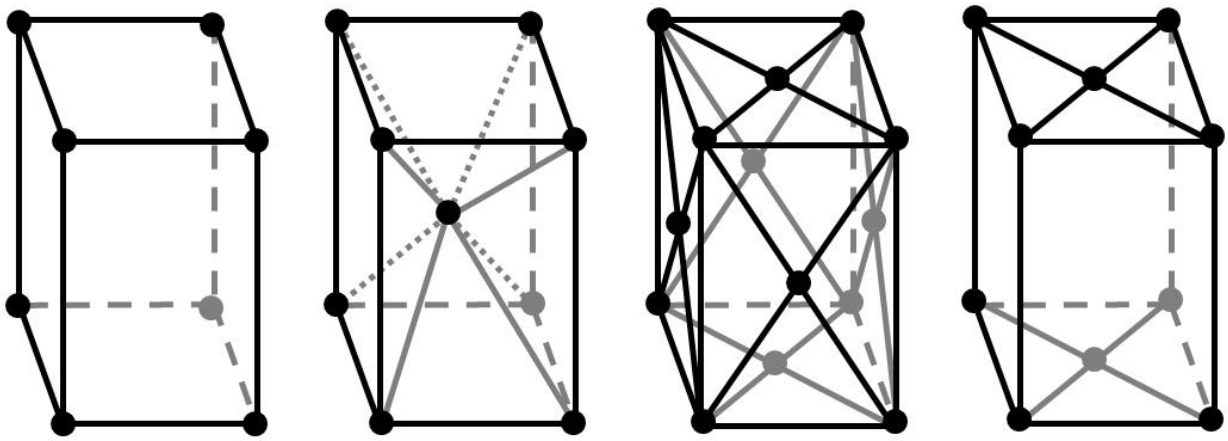


Figure 5: Primitive, body-centered, face-centered, and base-centered Bravais lattices.

1-7: Primitive (P). In a *primitive* lattice, lattice points lie at the corners of the unit cells. The seven crystal systems in the previous section are all examples of primitive unit cells, so you have already built seven of the first fourteen members of the Bravais lattices. In the space groups, primitive lattices are typically represented with the letter ***P*.**

8-10: Body centered (I). A *body-centered* crystal contains a lattice point at the center of the unit cell. In space groups, crystals with body-centered lattices are typically represented with the letter ***I*.** Body centered lattices are observed in the **cubic, tetragonal**, and **orthorhombic** crystal systems. Build one of each. Photograph or sketch the results.

11-12: Face-centered (F). A *face-centered* crystal contains a lattice point in each face of the unit cell. In space groups, crystals with body-centered lattices are typically represented with the letter ***F***. Face centered lattices are observed in the **cubic** and **orthorhombic** crystal systems. Build one of each. Photograph or sketch the results.

13-14: Base-centered. A *base-centered* crystal contains a lattice point in only one set of opposing faces of the unit cell - by convention, usually the *ab* face. In space groups, crystals with base-centered lattices are represented with the capital letter of the axis opposite the centered faces - usually the c-axis, so typically the letter ***C***. Base centered lattices are observed in the **orthorhombic** and **monoclinic** crystal systems. Build one of each. Photograph or sketch the results.

1. **Figures 1 and 5** drawn by Joy Heising after consulting original PNGs of orthorhombic unit cells by Daniel Mayer, traced in Inkscape by User:Stannered (Crystal stucture) [GFDL (http://www.gnu.org/copyleft/fdl.html) or CC-BY-SA-3.0 (http://creativecommons.org/licenses/by-sa/3.0/)], via Wikimedia Commons. Found at http://en.wikipedia.org/wiki/Bravais_lattice (accessed May 18, 2015).

2. Hahn, Th, Ed. *International Tables for Crystallography, Volume A: Space Group Symmetry*, 5th ed.; Springer-Verlag: New York, 2002.

Follow-up Questions

Q1. The mineral purpurite is a mixed iron/manganese phosphate which forms crystals in the space group *Pmna*, lattice parameters *a* = 5.8237 Å, *b* = 9.766 Å, *c* = 4.7771 Å.

a) To which crystal system is purpurite likely to belong? (circle):

cubic **tetragonal** **orthorhombic** **hexagonal** **monoclinic** **triclinic**

b) To which Bravais lattice does purpurite belong? (circle):

primitive **body centered** **base centered** **face centered**

Q2. For a face-centered cubic model with *a* = 10.0", find the distance between a face lattice point and an adjacent corner lattice point.

Q3. For a body-centered cubic model with *a* = 10.0", find the distance between the body-centered lattice point and a corner lattice point.